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SITE INDEX AND MAXIMUM GROSS YIELD CAPABILITY EQUATIONS FOR PONDEROSA PINE IN THE BLACK HILLS

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ABSTRACT

A mathematical model capable of use with computers or sophisticated calculators is derived from site index curves commonly used in management of ponderosa pine in the Black Hills. A second equation permits conversion of site index estimates to a maximum gross yield capability rating expressed in cubic feet of wood per acre per year.

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Site index (height of the dominant stand at a base age) has been used for many years to estimate site quality (Husch 1963). Much of the site index information still in use was developed years ago and presented in the form of tables and graphs which are not readily usable in computer programs (Brickell 1970). Hornibrook's (1939b) site index curves for ponderosa pine (*Pinus ponderosa* Laws.) in the Black Hills are still accepted as adequate for use in stands which have site trees 70 years old, or older, with class B crowns. It is desirable, therefore, to have a mathematical model of Hornibrook's site curves which can be used with today's electronic computers and calculators capable of being programed.

It is also worthwhile to have the means of converting a site index estimate to a productivity rating expressed in cubic feet of wood per acre. Forest Survey has for some time used yield capability to rate the wood-producing potential of forest land. The yield capability measure used for this purpose is defined as "...mean annual increment of growing stock attainable in fully stocked natural stands at the age of culmination of mean annual increment."

A problem with the use of this definition in the Black Hills and similar areas is that very few "natural stands" exist. Most stands have received repeated silvicultural treatments, and the density of such stands is far below the density of stands upon which most existing normal yield tables were based. Neither density may result in the

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site achieving maximum productivity. Also, in natural stands a portion of the site's production is lost to mortality, and this loss is not considered in Forest Survey's conventional definition of yield capability. This can lead to unrealistic estimates of potential productivity. Yield capability estimates based on normal yield tables are also influenced by utilization standards implicit in the tables. Such standards vary from one set of yield tables to another, and in any case may not reflect the site's full fiber production potential.

To avoid these problems the site's potential to yield wood should be expressed as the maximum amount of wood fiber producible on the natural site, regardless of stand size and stocking. A "natural" site is one that has not been modified through fertilization, irrigation, or the planting of genetically superior trees.

"Maximum gross yield capability" would be based on the culmination of mean annual gross increment in total stemwood cubic volume per acre. Total cubic mean annual gross increment is defined as the sum of the total stem cubic volume of all trees in the residual stand, at a specified age, plus the accumulated total stem cubic volume of all trees removed by thinning, mortality, or both, divided by stand age. Separate relationships could then be developed that would tie the site's maximum gross yield capability and stand density to the yield of the desired end product. Myers' (1971) computer program, PONYLD, which produces yield tables for managed ponderosa pine stands in the Black Hills, has made it possible to develop a maximum gross yield capability equation for that area.

The site index and maximum gross yield capability equations presented here were developed using Matchacurve techniques (Jensen 1964; Jensen and Homeyer 1970, 1971). These procedures were used because in both cases the task was to find an equation to describe a set of essentially smooth curves. Both models have been tested on a Monroe 1666[1] calculator so they should be usable with other calculators having similar capabilities.

Stocking levels that resulted in maximum gross yield capability are not recommended as the best stocking levels for management. Many other factors must be considered in management decisions.

SITE INDEX EQUATION

Hornibrook's site index curves are anamorphic, with a base age of 100 years, and are presented for site indices 40, 50, 60, 70, and 80. Average total age and average total height, in feet, for vigor class B trees, 70 years old or older, were needed data for determining site index. Hornibrook (1939a) modified Keen's tree classification for the Black Hills to define vigor class B trees as ones with "fair to moderately vigorous crowns with average width or narrower, and length less than 55 percent of the total height; either short wide crowns or long narrow ones, but not sparse or ragged; may be flat on one side, position either dominant or codominant but sometimes isolated."

[1]Use of trade or firm names is for reader information only, and does not constitute endorsement by the U. S. Department of Agriculture of any commercial product or service.

The site index equation developed from Hornibrook's curves is:

$$S = \frac{H}{X} - 0.3846$$

where

$X = -0.5234 + 1.8234 \cdot \exp[-[1.0989 - 0.006105(A)]^{2.35}]$

S = Site index, base age 100 years

H = Average total height of four to six vigor class B trees, in feet

A = Average total age of four to six vigor class B trees in years (if age is greater than 180 years, use 180 years in the equation)

A visual check of this model was made by plotting the predicted site index curves and comparing them to Hornibrook's site index curves. The curves were very close to each other, never differing more than one site index unit. Hornibrook's curves were drawn for ages from 70 to 360 years and for heights between 25 and 100 feet. Therefore, extending the use of this equation is not recommended. There is no assurance that the trends presented by Hornibrook can be extended beyond his curves.

MAXIMUM GROSS YIELD CAPABILITY EQUATION

Myers' PONYLD yield program allows the user to specify: (1) Initial values of stand age, mean stand diameter, and number of trees per acre; (2) maximum growing stock level to be maintained after initial or intermediate thinnings, once a mean stand diameter of 10 inches has been reached; (3) the length of time between thinnings; (4) the stand's site index; and (5) other items not of primary importance in this analysis. This flexibility allows a very large number of combinations of these variables to be tested to determine which combination would produce maximum gross yield capability. To reduce the number of combinations to a reasonable level, some assumptions were made.

First, only site index values from 30 to 100, in increments of 10, were examined. This assured coverage corresponding to the range of Hornibrook's site curves.

Second, the initial values of stand age, mean stand diameter, and number of trees per acre for each site index level were either equated to or extrapolated from the quantities given by Myers (1966). These values are probably representative of stand conditions in Black Hills ponderosa pine.

Finally, the thinning cycle was tested only for values of 10, 20, and 30 years. These seemed to cover a reasonable range of cutting cycles for the Black Hills.

With these assumptions, maximum growing stock level was then screened for each cutting cycle to determine which combination produced maximum gross yield capability at each site index level. PONYLD was modified by the author to include total cubic mean annual increment within the program.

In the first screening, the initial thinning level was set equal to the stocking level for the intermediate thinnings. This was done to reduce the range of values examined in the next step. The second screening allowed the initial thinning level to change as well as the intermediate thinning level. The results of these screening runs indicated that total cubic mean annual increment was maximized when the initial and intermediate stocking levels were equivalent. They also showed that the calculated mean annual increments departed somewhat from the expected values.

To help smooth out the irregularities, the maximum gross yield capability was picked for each stocking level-cutting cycle-site index combination. The age and stocking level at which these maximum gross yield capabilities occurred were plotted over site index for each cutting cycle, and linear equations were developed.

These equations were then used to calculate a "smoothed" age and stocking level for each site index and cutting cycle combination. The appropriate maximum gross yield capabilities for these "smoothed" values were then picked from the tables and used as final values for the rest of the analysis.

Examination of the maximum gross yield capabilities for each cutting cycle indicated that the 20-year cycle produced slightly higher values, so it was used in the development of the equation:

$$MGYC = -32.47 + 203.97 \cdot \exp\left[-[1.8117 - 0.01647(S)]^{1.024}\right]$$

where

MGYC = Maximum gross yield capability in cubic feet per acre per year

S = Site index, base age 100 years

The use of this equation should be restricted to site indices ranging from 30 to 100. In tabular form, maximum gross yield capability for these site indices is:

Site index	*Maximum gross yield capability* *ft³/acre/yr*
30	21.67
40	31.67
50	43.48
60	57.39
70	73.77
80	93.01
90	115.53
100	141.74

For the 20-year cutting cycle, the relationship of site index to age and stocking level of MGYC was found to be:

$$A = 210 - S$$

$$St = 80 + S$$

where

A = Age, in years, at which gross yield capability maximized

S = Site index, base age 100 years

St = Stocking level, in square feet of basal area per acre (after mean stand diameter reaches 10 inches) at which gross yield capability maximized

It should be remembered that these stocking levels are not being recommended as the best stocking levels for management. Rather, they are the stocking levels at which maximum gross yield capability was indicated.

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